

INTERPRETATION AND COMPARISON OF
OUTPUT SIGNALS FROM LINEAR-FM
PULSE-COMPRESSSION SYSTEMS

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-64-129

AUGUST 1964

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FOREWORD

The author wishes to thank Mr. R.W. Jacobus for many helpful discussions and Mrs. P.J. Gassler for her programming efforts.

INTERPRETATION AND COMPARISON OF OUTPUT SIGNALS
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ABSTRACT

Several expressions for the output time function of a linear-FM pulse-compression filter, which have recently been derived, (ESD-TDR-64-128) are compared and interpreted. Comparisons are made analytically as well as graphically with respect to several pertinent parameters.

It is shown that if the phase and envelope measurements of signals returned from an actual target were to be interpreted as though the signal had undergone a Doppler shift (instead of a time dilation), a considerable error would result for high target velocities.

The feasibility of using an all-pulse-compression system for obtaining accurate estimates of radar target parameters is demonstrated. Simple analytic expressions, which describe the phase and envelope quite well during the time of measurement, are presented, and a procedure for properly interpreting the data is indicated.

REVIEW AND APPROVAL

This technical documentary report has been reviewed and is approved.



HARRY BYRAM
Acting Chief, Radar Division
Directorate of Radar and Optics

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INTERPRETATION AND COMPARISON OF OUTPUT SIGNALS FROM LINEAR-FM PULSE-COMPRESSSION SYSTEMS

This document is an extension of a recent theoretical investigation^{[1]*} in which several expressions (with different degrees of accuracy) for the output of a linear-FM pulse-compression system were derived. The purpose of this report is to compare and interpret these expressions. For convenience, we shall first give a brief summary of Ref. [1] , using the same notation with only a few exceptions.

The transmitted signal is assumed to have a flat band-limited amplitude spectrum and a linear group delay, as illustrated in Fig. 1. For simplicity, only the positive half of the spectrum is shown. (Here, $\omega = 2\pi f$, as usual.) For large values of the time-bandwidth product, $TW/2\pi$, this is approximately a linear FM signal. The following two pulse-compression systems are of particular interest here:

| Pulse-Compression Systems | $B = W/2\pi$ | T | $f_o = \omega_o/2\pi$ |
|----------------------------|---------------------|------------------------|-----------------------|
| No. 1 (1000:1 system) | 10^6 cps | 10^{-3} cps | 1280 Mc |
| No. 2 (10,000:1 system) | 5×10^6 cps | 2×10^{-3} sec | 1280 Mc |

*Numbers in brackets designate references at end of text.

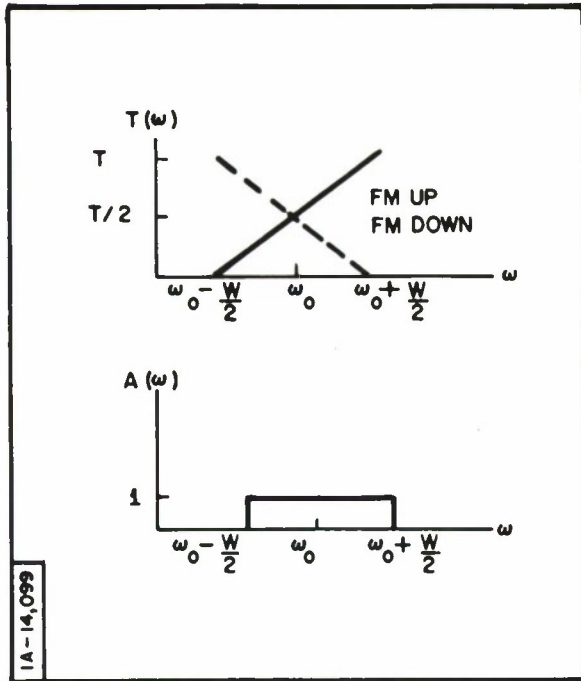


Fig. 1. Transmitted Signal

The pulse-compression system model in Fig. 2 is used in the analysis.

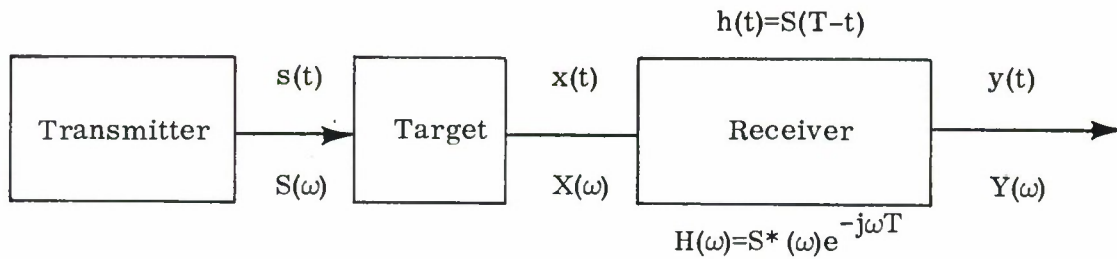


Fig. 2. Model of Pulse-Compression System

The receiver is seen to be matched to the transmitted signal. The target is assumed to be a point target. For a target moving with uniform radial velocity, V , the relationship between the transmitted spectrum, $S(\omega)$, and the received spectrum, $X(\omega)$, is

$$X(\omega) = k \left(\frac{c+V}{c-V} \right) S \left(\frac{c+V}{c-V} \omega \right) e^{-j\omega \frac{2R}{c}}, \quad (1)$$

- k is a constant which depends on the target's cross section,
R is the target's range when the pulse strikes it,
V is the target's radial velocity when the pulse strikes it, and
c is the velocity of light in free space.

If the change in range and radial velocity due to the target's radial acceleration (and higher order terms) during the illumination time is small, Eq. (1) is a good approximation. This is assumed to be so for cases of interest to us.

Thus, the original spectrum, $S(\omega)$, has become attenuated, delayed, and distorted. We may think of each frequency component, ω_i , as having been shifted to a new value, $\omega_i \left(\frac{c-V}{c+V} \right) = \omega_i \left(1 - \frac{2V}{c+V} \right)$. The amount of shift depends both on the velocity of the target and on the particular frequency of each component. In the time domain this corresponds to a "time dilation."

For a narrow-band signal, all frequency components are close to the center frequency ω_0 and are, therefore, shifted approximately by the same amount, $\frac{2V}{c+V} \omega_0 \left(\approx \frac{2V}{c} \omega_0 \text{ for } V \ll c \right)$. Under this assumption, the whole spectrum is merely shifted and no distortion has taken place. Since

$\frac{c+V}{c-V} \approx 1$, Eq. (1) then simplifies to

$$X(\omega) = kS \left(\omega + \frac{2V}{c} \omega_0 \right) e^{-j\omega \frac{2R}{c}} . \quad (2)$$

We concern ourselves here with radial velocities up to 10,000 meters/second.

The output time function (refer to Fig. 2) is expressed in the form

$$y(t) = E \cos \theta .$$

Assuming that a simple Doppler shift is an adequate representation [i. e. , assuming that (2) is valid] , one obtains

$$E = E_d = \frac{2k}{\pi} \left(\frac{W'}{2} \right) \frac{\sin t' \frac{W'}{2}}{t \frac{W'}{2}} , \quad (3a)$$

$$\theta = \theta_d = \omega_0 t' + \delta , \quad (3b)$$

where the subscript d denotes the Doppler case. Here

$$\begin{aligned} \frac{W'}{2} &= \frac{W}{2} - \frac{V}{c} \omega_0 , \quad \omega_0' = \omega_0 \left(1 - \frac{V}{c} \right) , \\ t' &= t - \frac{2R}{c} - T \mp \frac{T}{W} \left(\frac{2V}{c} \omega_0 \right) \begin{cases} - \text{ for FM up} \\ + \text{ for FM down} \end{cases} , \\ \delta &= \frac{T}{W} \left(\frac{2V}{c} \omega_0 \right) \left[\pm \omega_0' - \frac{W}{2} \right] \begin{cases} + \text{ for FM up} \\ - \text{ for FM down} \end{cases} . \end{aligned}$$

where the \pm and \mp notations imply that the first sign is for FM up, and the second sign is for FM down. Note that $t' = 0$ corresponds to the maximum value of E . The above expressions were derived primarily as a check and as an interesting comparison with the expressions that follow.

Using Eq. (1) as a description of the returned spectrum, one obtains, without any further approximations,

$$E_e = \frac{2k}{\pi} \frac{c+V}{4} \sqrt{\frac{\pi W}{TVc}} \sqrt{[S(u_1) - S(u_2)]^2 + [C(u_1) - C(u_2)]^2}, \quad (4a)$$

$$\theta_e = \pm \left\{ \frac{\left[t'' \pm \frac{T}{W} \frac{4Vc}{(c-V)^2} \omega_0'' \right]^2}{2 \frac{T}{W} \frac{4Vc}{(c-V)^2}} - \tan^{-1} \frac{S(u_1) - S(u_2)}{C(u_1) - C(u_2)} \right\}, \quad (4b)$$

where the subscript e denotes the exact case. The terms $S(u)$ and $C(u)$ are the Fresnel integrals, as defined by Eq. (40a) in Ref. [1]. Here we have

$$u_1 = \frac{c-V}{2} \sqrt{\frac{W}{\pi TVc}} \left[\frac{W''}{2} \left(\frac{T}{W} \right) \frac{4Vc}{(c-V)^2} \mp t'' \right],$$

$$u_2 = \frac{c-V}{2} \sqrt{\frac{W}{\pi TVc}} \left[- \frac{W''}{2} \left(\frac{T}{W} \right) \frac{4Vc}{(c-V)^2} \mp t'' \right],$$

$$\frac{W''}{2} = \frac{W}{2} \left(\frac{c}{c+V} \right) - \omega_0 \frac{V}{c+V}, \quad \omega_0'' = \omega_0 \left(\frac{c}{c+V} \right) - \frac{W}{2} \left(\frac{V}{c+V} \right),$$

$$t'' = t - \frac{2R}{c} - T \pm \frac{2V}{c-V} \left(\frac{T}{W} \right) \omega_0 - \frac{W}{2} - \frac{2c}{c-V} \omega_0''.$$

The point $t'' = 0$ corresponds to the peak of the envelope. Approximations to (4a) and (4b) were obtained using asymptotic expansions of the Fresnel integrals.

Denoting these approximations by the subscript a, we have

$$E_a = \frac{2k}{\pi} \left(\frac{c+V}{c-V} \right) \frac{W''}{2} \frac{\sin t'' \frac{W''}{2}}{t'' \frac{W''}{2}} \quad (5a)$$

$$\theta_a = \omega_0'' t'' \pm \frac{2Vc}{(c-V)^2} \left(\frac{T}{W} \right) \left[(\omega_0'')^2 - \left(\frac{W''}{2} \right)^2 \right]. \quad (5b)$$

The maximum value of E_a also occurs when $t'' = 0$. A second method for approximating the output function was used. A simplification was made with the output spectrum, thus eliminating the Fresnel integrals. This method yielded an expression for E identical to (5a) and an expression for θ which was equal to (5b), except that the $\left(\frac{W''}{2}\right)^2$ term was missing. Although $\left(\frac{W''}{2}\right)^2 \ll (\omega_0'')^2$, we shall see later that this term is necessary to properly interpret the expressions for θ_e . Equations (5a) and (5b) are expected to be good approximations to the exact expressions for the values of time which satisfy

$$|t''| > 100 W'' \frac{Vc}{(c-V)^2} \left(\frac{T}{W}\right) \approx 100 \frac{V}{c} T. \quad (6)$$

One comment should be made at this time. In Ref. [1] we referred to the expressions denoted here by E_d , E_e and E_a somewhat loosely as envelope. Strictly speaking, this term applies only to E_e , since E_d and E_a take on negative values. The envelopes corresponding to Eqs. (3) and (5) are $|E_d|$ and $|E_a|$, respectively. The abrupt phase reversal which occurs as a result of the change in sign for the amplitude can be absorbed in the phase function by adding a term $n\pi$, where n is even when the amplitude is positive, and odd when the amplitude is negative. We shall later compare E_e with $|E_a|$ and θ_e with the appropriately adjusted function θ_a .

Instead of using the exact but unwieldy expressions in (4), we would like to use the approximate expressions in (3) or (5), perhaps in conjunction with a simple (we hope) error function.

Let us first compare Eqs. (3a) and (5a). They are both of the form $\sin x/x$. One immediate observation is that the peaks occur at different times. In the simple Doppler case (Eq. 3a), the peak occurs when $t' = 0$, i. e., when

$$t = t_d = \frac{2R}{c} + T \pm \frac{T}{W} \left(\frac{2V}{c} \omega_0 \right). \quad (7)$$

(We use t_d to denote the time of the peak for the Doppler case.) The peak of the approximate expression (5a) coincides with that of the exact expression.

This occurs when $t'' = 0$, i. e., when

$$t = t_e = \frac{2R}{c} + T \pm \frac{T}{W} \left(\frac{2V}{c-V} \right) \left[\omega_0'' \left(\frac{2c}{c-V} \right) - \omega_0 + \frac{W}{2} \right]. \quad (8)$$

Note that if the velocity is constant, true estimates of range and velocity can be obtained by alternately transmitting FM up and FM down and taking the sum and difference, respectively, of the time delays. This is true for both (7) and (8).

Letting $\Delta t = t_e - t_d$ and substituting for ω_0'' , we obtain

$$\begin{aligned} \Delta t &= \pm \frac{T}{W} \left(\frac{2V}{c-V} \right) \left[\omega_0 \frac{V}{c} + \frac{2V^2}{c^2 - V^2} + \frac{W}{2} \left(\frac{c-V}{c+V} \right) \right] \\ &\approx \pm \frac{T}{W} \left(\frac{2V}{c} \right) \left[\omega_0 \frac{V}{c} + \frac{W}{2} \right]. \end{aligned} \quad (9)$$

At the maximum velocity being considered here ($V = 10^4$ meters/second), we have

$$\Delta t \approx \pm 0.0362 \mu\text{sec. for system No. 1}$$

$$\Delta t \approx \pm 0.0678 \mu\text{sec. for system No. 2}$$

This corresponds to a range difference of approximately 46 half-wavelengths for system No. 1 and 87 half-wavelengths for system No. 2.

With the width of the compressed pulse being approximately 1 and 0.2 $\mu\text{sec.}$, respectively, the difference in delay above corresponds to about 4 percent of the pulse width for system No. 1 and 34 percent of the pulse width for system No. 2.

Except for this difference, expressions (3a) and (5a) are virtually the same since $\frac{c+V}{c-V} \approx 1$ and $W'' \approx W'$ for $V \leq 10^4$ meters/second.

For a $\sin x/x$ function, the distance between the points where $x = \pm \pi/2$ is commonly referred to as the pulse width. Denoting the pulse width thus defined by τ_d and τ_a (with reference to (3a) and (5a), respectively), and letting $B = W/2\pi$ and $f_0 = \omega_0/2\pi$, we have

$$\tau_d = \frac{1}{B'} = \frac{1}{B - \frac{2V}{c} f_0} = \frac{c}{Bc - 2f_0 V} \quad , \quad (10a)$$

$$\tau_a = \frac{1}{B''} = \frac{1}{B \left(\frac{c}{c+V} \right) - f_0 \left(\frac{2V}{c+V} \right)} = \frac{c+V}{Bc - 2f_0 V} \quad . \quad (10b)$$

Expression (10b) is correct for a receding target. For an approaching target, the bandwidth of the returned signal would be widened and we would get a narrower pulse with a pulse width

$$\tau_a = \frac{c-V}{Bc - 2f_0 V} \quad (10c)$$

Expression (10a) is valid in either case. Thus, $\tau_a > \tau_d$ for a receding target, but $\tau_a < \tau_d$ for an approaching target, the difference being equal to

$$|\tau_a - \tau_d| = \frac{V}{Bc - 2f_0 V} \quad .$$

This is only about 0.003 percent of the pulse width in the worst case.

Zeros of the two $\sin x/x$ functions occur at $t' = \frac{n}{B'}$ and $t'' = \frac{n}{B''}$,

respectively. (Here $n = 1, 2, 3, \dots$.) It is seen that the two envelopes are about the same. In both cases the pulse gets wider as V increases, while the peak amplitude decreases in the same proportion.

The two approximate expressions for the phases, θ_d and θ_a are easily compared. Both are linear functions of time with approximately the same slope (ω'_0 and ω''_0 , respectively). The value of θ_d at the peak of the envelope (i. e. , at $t' = 0$) is

$$\theta_d (t' = 0) = \frac{T}{W} \left(\frac{2V}{c} \omega_0 \right) \left(\pm \omega'_0 - \frac{W}{2} \right) \quad (11)$$

Similarly, at $t'' = 0$ we have, from (5b)

$$\theta_a(t'' = 0) = \pm \left(\frac{T}{W} \right) \frac{2Vc}{(c-V)^2} \left[(\omega_0'')^2 - \left(\frac{W''}{2} \right)^2 \right] \quad (12)$$

These values are substantially different from each other, as one would expect from the difference in the time delays.

Let us next consider the exact equations (4a) and (4b). It can be shown that the envelope E_e has even symmetry with respect to the line $t'' = 0$. We established in Ref. [1] that $\frac{d}{dt} E_e = 0$ at $t'' = 0$, from which we concluded that the peak occurs at $t'' = 0$. It can also be shown that $\frac{d}{dt} E_e = 0$ whenever $t'' = \frac{n}{B''}$, where $n = 1, 2, 3, \dots$ and $B'' = W''/2\pi$. Thus, not only does the peak of E_e occur at the same time as the peak of E_a , but all the minimum values of E_e occur exactly at the same times as the zero values of E_a . This implies that the approximation might be better than we had expected, and that it might even apply near the peak of the envelope.

With the hope that this is the case both for the envelope and the phase approximations, a computer program was written to compute $\Delta\theta = \theta_e - \theta_a$ (modified θ_a) and $\Delta E = E_e - |E_a|$. Since these differences are expected to be largest for high radial velocities, numerical answers were obtained for $V = 10^3$ and $V = 10^4$ meters/second for both the No. 1 and No. 2 systems.

Power series expansions were used for the Fresnel integrals. These expansions converged for some values of the arguments and diverged for others.

Where divergence occurred, the desired answers were obtained by hand calculations with the aid of tables.

The difference in the envelopes was normalized with respect to the peak value of E_a at each particular velocity, i. e., we obtained

$$\Delta E' = \frac{\Delta E}{(E_a)_{\max}} = \frac{E_e}{(E_a)_{\max}} - \frac{|E_a|}{(E_a)_{\max}} = E'_e - E'_a \quad (13)$$

Using (4a) and (5a), and letting $B = W/2\pi$, $f_0 = \omega_0/2\pi$, we have

$$\begin{aligned} \Delta E' = E'_e - E'_a = & \frac{c-V}{B'} \sqrt{\frac{B}{8TVc}} \sqrt{[S(u_1) - S(u_2)]^2 + [C(u_1) - C(u_2)]^2} \\ & - \left| \frac{\sin \pi t' B'}{\pi t' B'} \right| \end{aligned} \quad (14)$$

The values of $\Delta E'$, E'_e and E'_a thus obtained were plotted as a function of t' . We refer to E'_e as the exact envelope (normalized and to E'_a as the $|\sin x/x|$ envelope.

Curves of E'_a and $\Delta E'$ versus t' are shown in Fig. 3 for $V = 10^3$ and $V = 10^4$ meters/second. In the figure, $\Delta E'$ is plotted on a 10x magnified scale. It can be seen that the difference between the exact and approximate expressions is quite small. The largest difference occurs when $E'_a = 0$ on either side of the main lobe.

It is interesting to see the effect of radial velocity on the pulse width. (The pulse width, as defined by (10), is half of the width of the main lobe). It is important to remember that each envelope was normalized with respect to

its own peak value, so that the decrease in the peak value does not show in the figure.

Figure 4 is similar to Fig. 3. It shows E'_a and ΔE as functions of t'' for the 10,000:1 system and for a radial velocity of 10^3 meters/second. The difference between E'_e and E'_a is still rather small.

Figure 5 depicts the worst case considered, system No. 2, for a target velocity of $V = 10^4$ meters/second. Now the exact envelope is sufficiently different from the approximate envelope to be noticeable when plotted on the same scale. It can be seen clearly in the figure that E'_e , the exact envelope, does not take on zero values but has minima at those points where E'_a , the approximate envelope, goes to zero. Again, the difference between the two envelopes is largest at the zero points closest to the main lobe.

We note that the overall behavior of E'_e approaches that of E'_a more and more closely as $|t''|$ increases. This is consistent with our theoretical predictions. Note that the approximations are quite good, even in the immediate vicinity of $t'' = 0$. This far exceeds our expectations.

Let us next consider the difference between the exact and the approximate phase expressions, $\Delta\theta = \theta_e - \theta_a$. We had originally (in Ref. [1]) ignored the term $\left(\frac{W''}{2}\right)^2$ in the expression for θ_a , and the numerical calculations were made on this basis; i. e., we computed

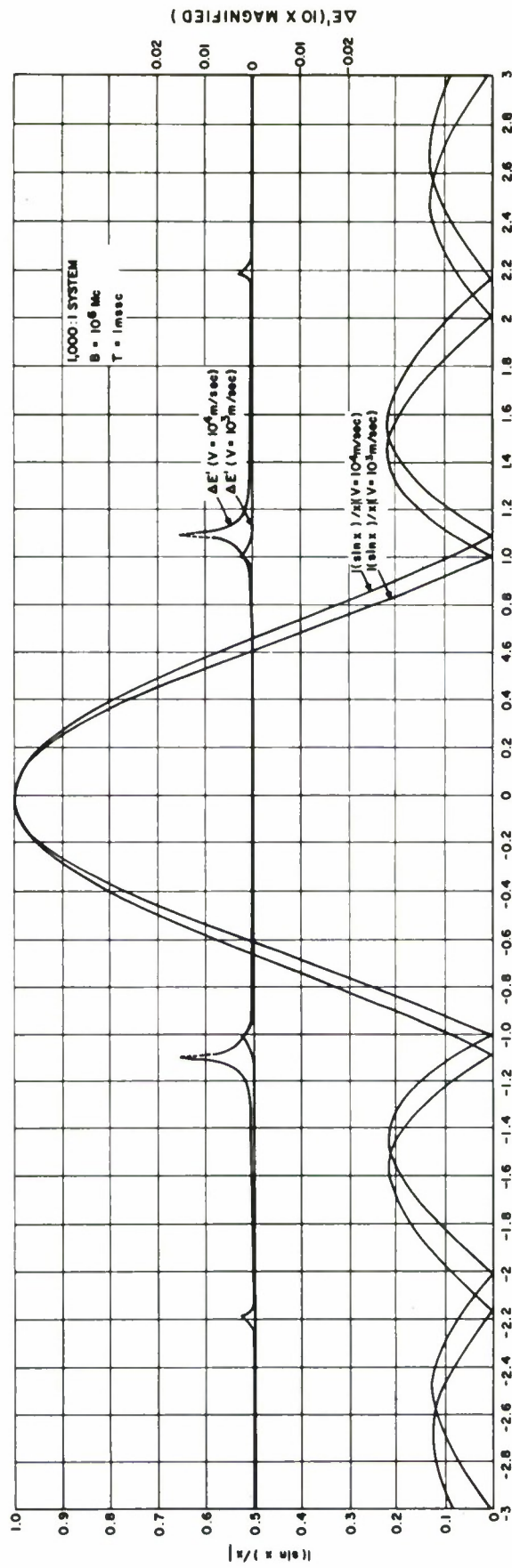


Fig. 3. $|(\sin x)/x|$ Envelope and $\Delta E'$ vs. t''

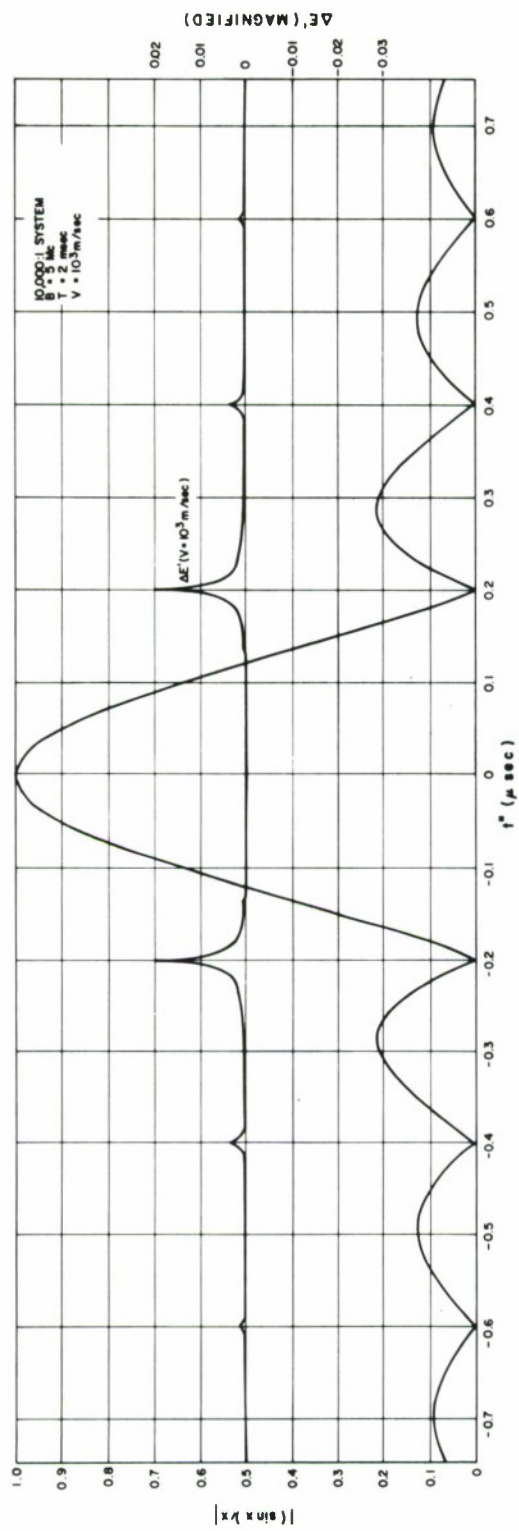


Fig. 4. $|\sin x|/x$ | Envelope and $\Delta E'$ vs. t''

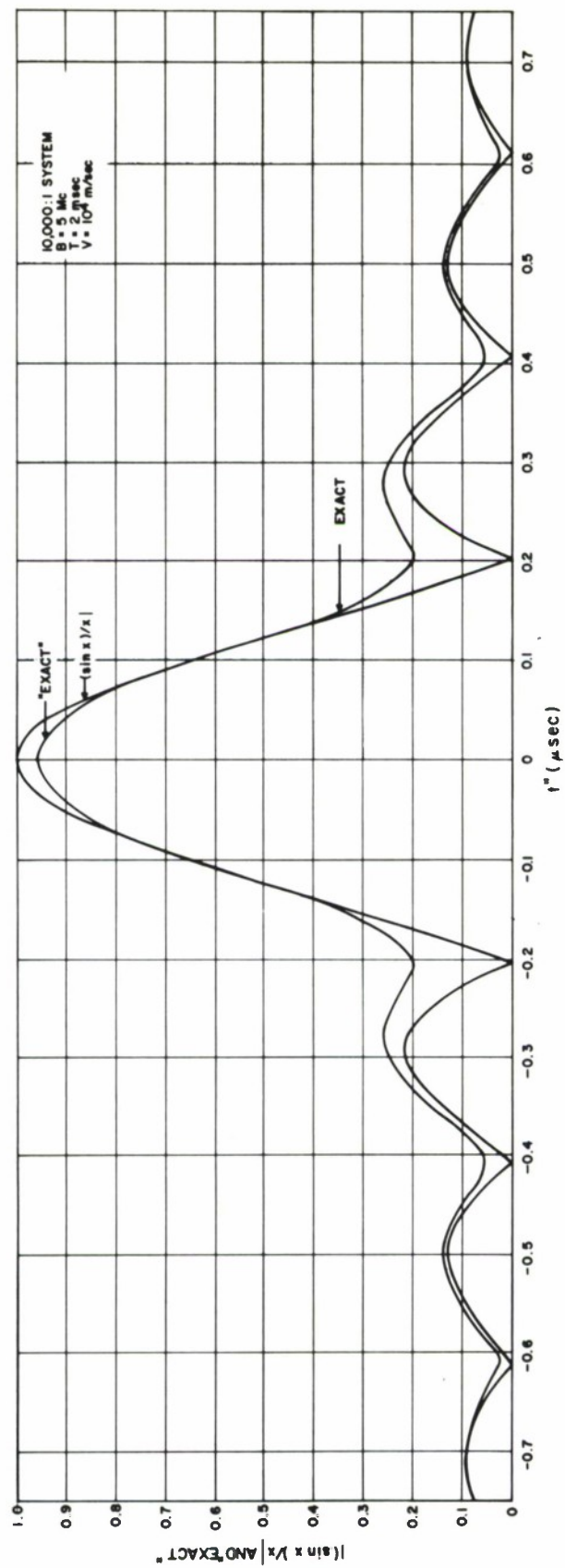


Fig. 5. $|(\sin x)/x|$ Envelope and "Exact" Envelope

$$\Delta\theta = \pm \left\{ \frac{W(c-V)^2}{8TVc} (t'')^2 - \tan^{-1} \left[\frac{S(u_1) - S(u_2)}{C(u_1) - C(u_2)} \right] \right\} . \quad (15)$$

In arriving at this expression, a θ_a was used which is the same as in (5b), except that the $\left(\frac{W''}{2}\right)^2$ term is missing. Since W'' is not a function of time, the $\Delta\theta$ obtained from (15) can readily be adjusted to account for this missing term.

It can be shown that both terms in (15) have even symmetry, so that $\Delta\theta$ need only be computed for positive (or negative) values of t'' . Figures 6 through 9 show graphs of $\Delta\theta$ versus t'' . As discussed before, θ_a was adjusted (by adding π radians at appropriate times) to account for the abrupt phase change which occurs when E_a changes sign ($|E_a|$ and E_e are never negative).

Figure 6 applies to system No. 1 with $V = 10^3$ meters/second. Note that the vertical scale has been expanded near $\Delta\theta = 0$ to show greater detail (Fig. 6a). The discontinuities in the graph may appear strange at first sight, but can be explained in the following way: θ_a makes a sudden jump of π radians at the points where $|E_a| = 0$ (i. e., at the times when E_a changes sign). This can be depicted by a step function, as shown. The exact phase function, θ_e , also goes through a phase change of π radians, but does it in a gradual fashion, as indicated. The difference, $\Delta\theta$, has the same shape as in Fig. 6.

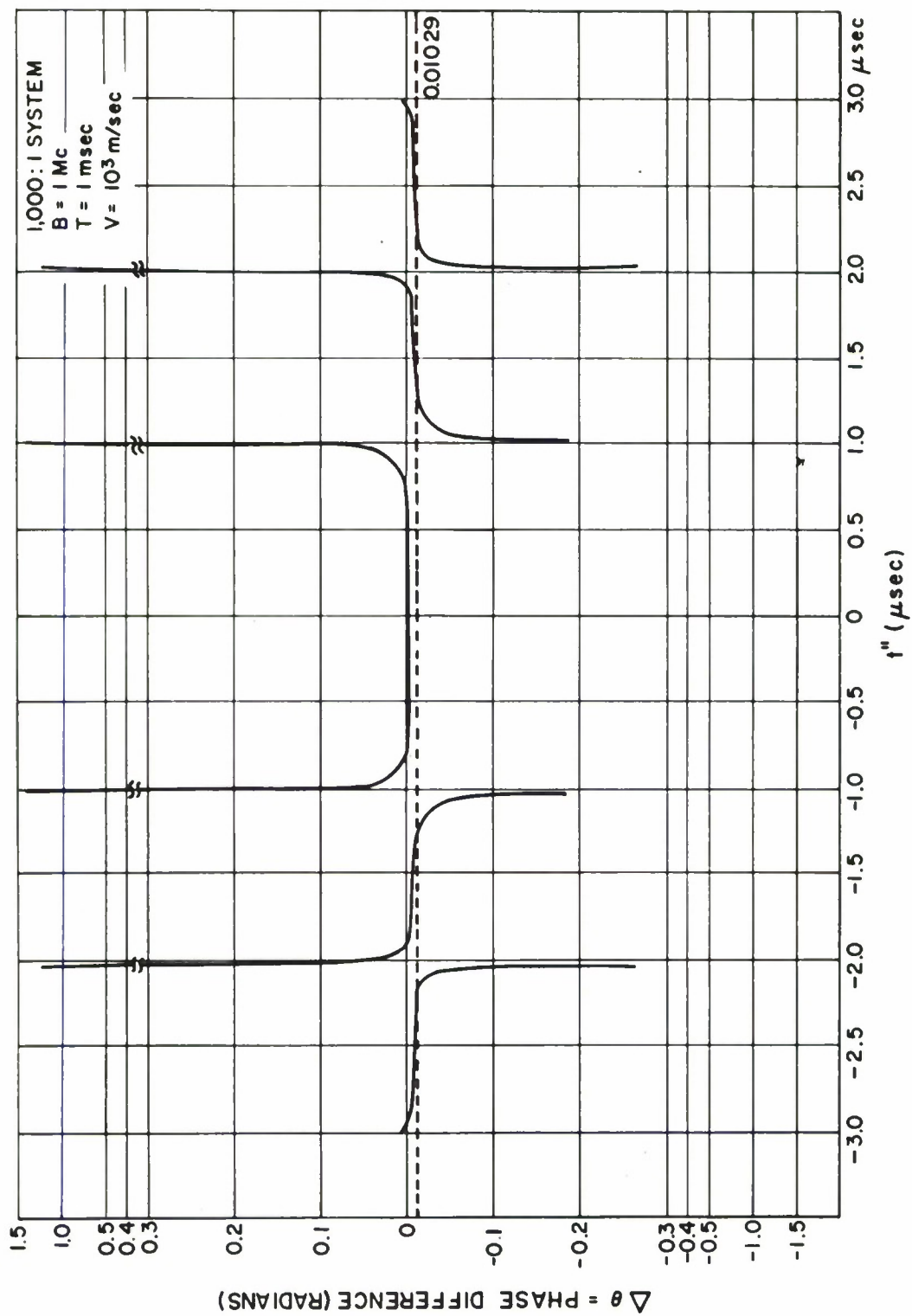


Fig. 6. Phase Difference vs. t''

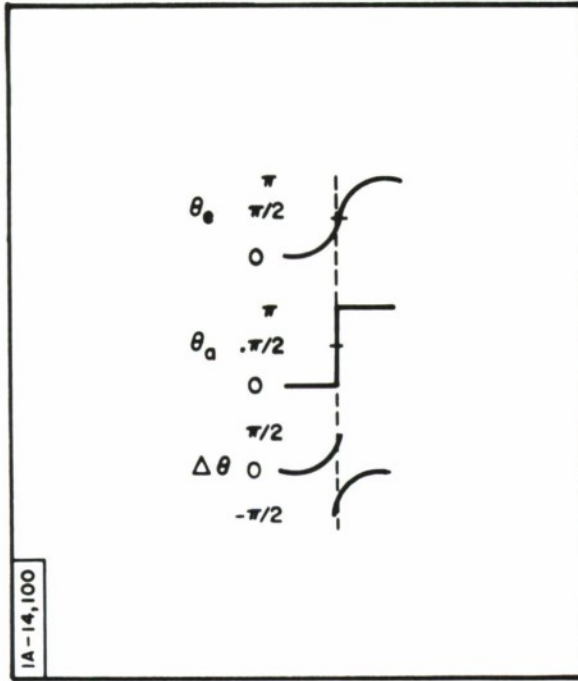


Fig. 6a. Expansion of Vertical Scale

Except for these discontinuities, $\Delta\theta$ is seen to approach the dashed line indicated in Fig. 6a. This dashed line would be the zero axis if we had not omitted the $\left(\frac{W''}{2}\right)^2$ term in Eq. (5b). It can be seen that the curve is quite flat in the vicinity of $t'' = 0$.

The same comments apply to Figs. 7, 8, and 9. Note that we are still using an expanded scale for Figs. 7 and 8, but not for Fig. 9. Figure 9 depicts the worst case, i. e., the 10,000:1 system with $V = 10^4$ meters/second. The difference between the abrupt and the gradual transitions is considerably greater than in the other cases. Even for this worst case, however, $\Delta\theta$ is fairly constant in the immediate vicinity of $t'' = 0$.

Let $\Delta\theta(0)$ be the value of $\Delta\theta$ in the flat region near $t'' = 0$. In each of the four cases we have considered it turned out that

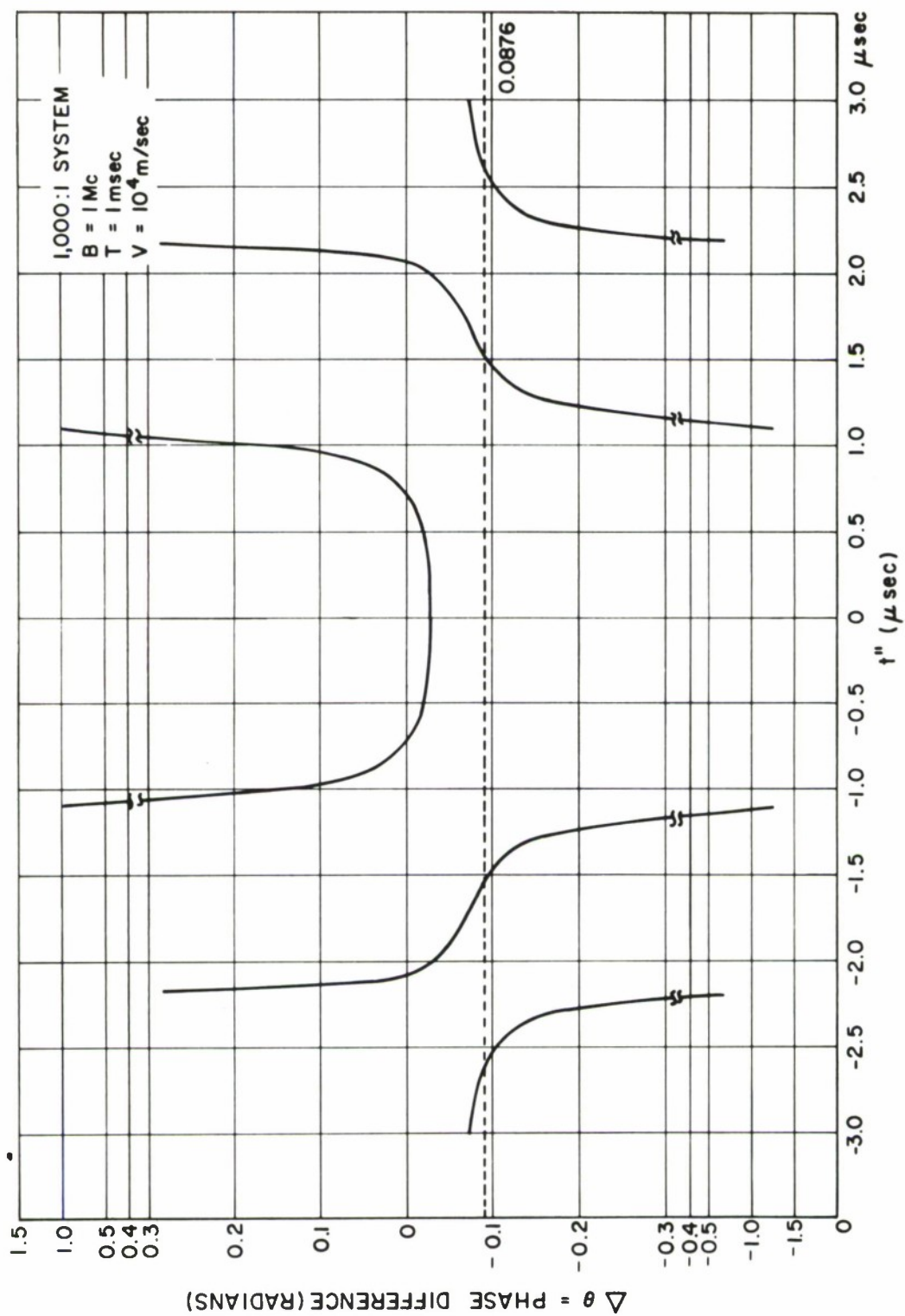


Fig. 7. Phase Difference vs. t''

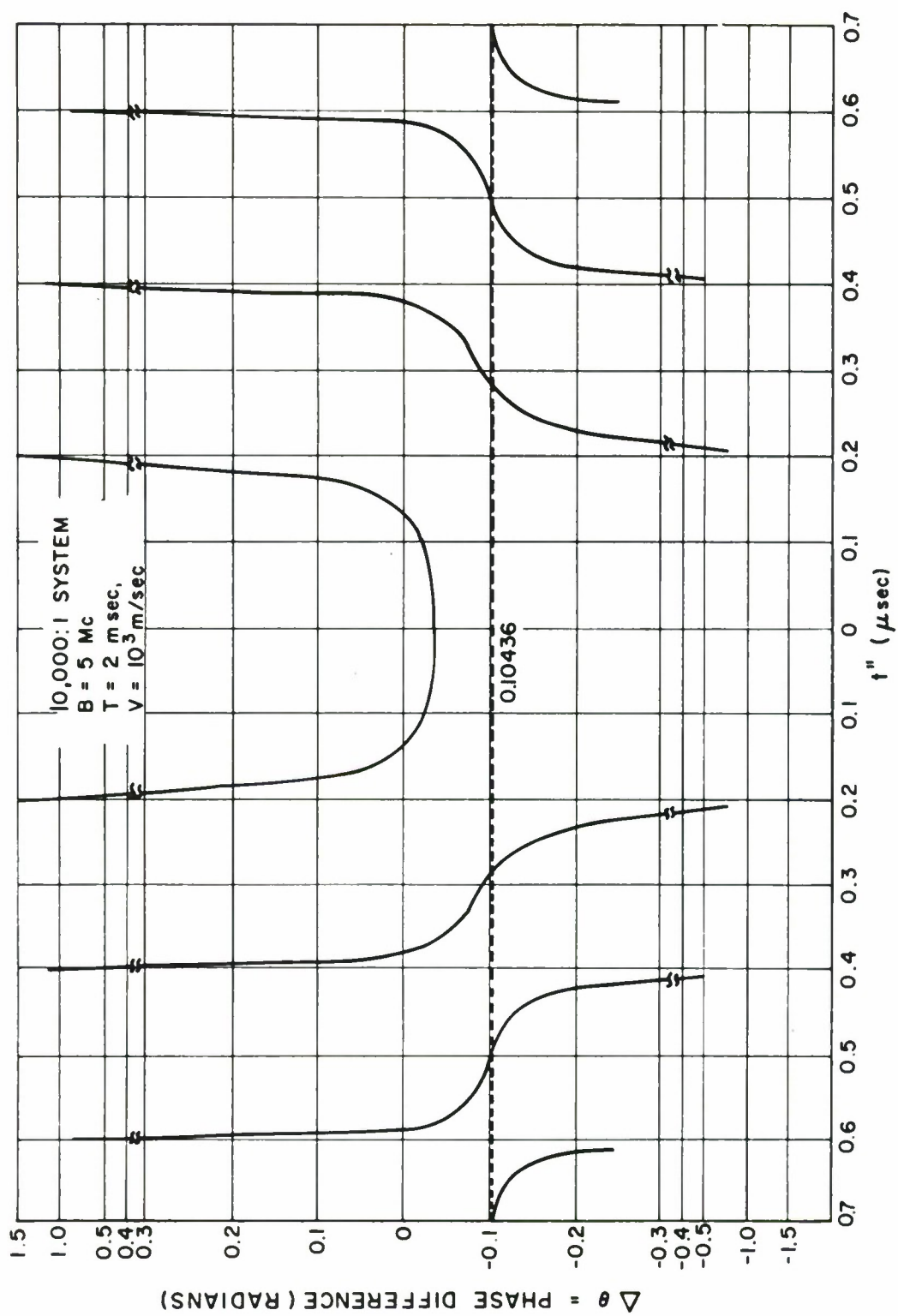


Fig. 8. Phase Difference vs. t''

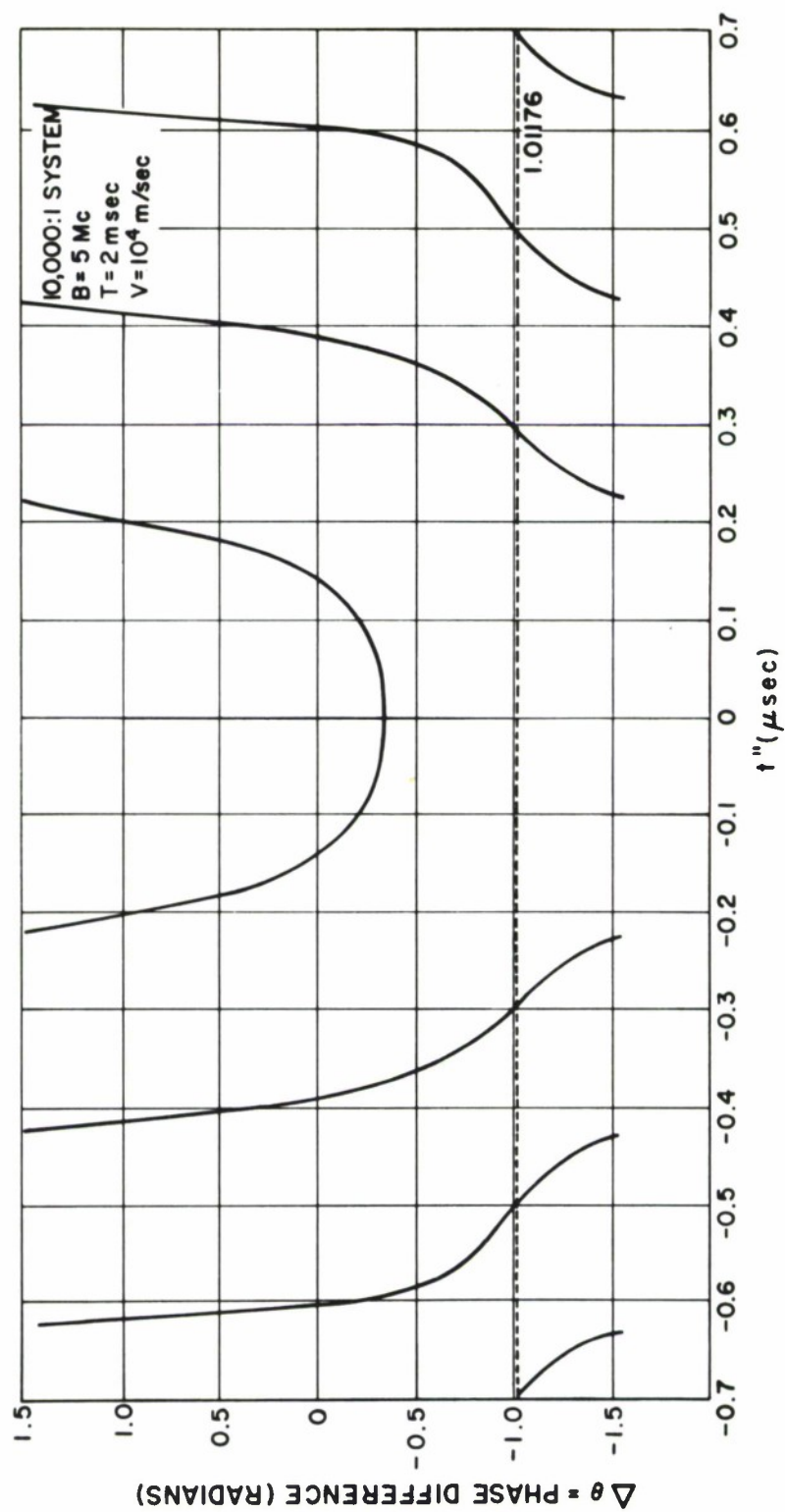


Fig. 9. Phase Difference vs. t''

$$\Delta\theta(0) = \frac{2}{3} \left[\left(\frac{W''}{2} \right)^2 - \frac{2TVc}{(c-V)^2 W} \right]. \quad (16)$$

This is $\frac{2}{3}$ times the $\left(\frac{W''}{2} \right)^2$ term in (5b), which was originally ignored. We shall assume that the empirical relation (16) will hold, in general, since the behavior of $\Delta\theta$ and ΔE has been quite consistent so far. We have found (16) to hold for large values of velocity ($V = 10^3$ and $V = 10^4$ meters/second). Both Eq. (16) and intuitive reasoning indicate that $\Delta\theta(0)$ is small for low velocities (say for $V \leq 100$ meters/second). Thus, even if (16) should not hold, in general, only a small error would result by assuming that it does. We shall, therefore, approximate the exact phase near $t'' = 0$ by using the approximate phase expression plus the above correction term; i. e., we let

$$\theta_e = \theta_a + \Delta\theta(0), \text{ for small values of } t''.$$

Using (5b), and (16), we obtain

$$\theta_e = \omega_0'' t'' + \frac{2TVc}{(c-V)^2 W} \left[(\omega_0'')^2 - \frac{1}{3} \left(\frac{W''}{2} \right)^2 \right]. \quad (17)$$

Even in the worst case (as depicted by Fig. 9), this expression is quite good

for values of $|t''| \leq \frac{1}{8} \tau$, where τ is the pulse width defined previously.

Thus, we can obtain very good point estimates of phase by integrating the phase over a symmetrical region (with respect to $t'' = 0$) which is not greater than $\tau/4$. Fortunately, this is exactly where we want to make our phase measurement, because the signal-to-noise ratio is highest near the peak of the envelope.

Consider a phase measurement technique similar in principle to that used in the sequential Doppler processor^[2]. Suppose we use a single reference oscillator of frequency f_0 , which is phase-locked to the transmitted signal; i. e., suppose we have available the reference signal $r_c(t) = A \cos \omega_0 t$, where A is a constant. Let us call $\omega_0 t$, the reference phase, θ_r so that

$$r_c(t) = A \cos \theta_r = A \cos \omega_0 t. \quad (18)$$

If we pass this signal through a 90-degree phase shifter, we obtain

$$r_s(t) = A \sin \theta_r = A \cos \left(\theta_r - \frac{\pi}{2} \right). \quad (19)$$

Let us mix (i. e., multiply) the output function, $y_e(t)$, with both $r_c(t)$ and $r_s(t)$.

We have

$$\begin{aligned} r_c(t) y_e(t) &= (A \cos \theta_r) (E_e \cos \theta_e) \\ &= \frac{AE_e}{2} [\cos(\theta_r - \theta_e) + \cos(\theta_r + \theta_e)]. \end{aligned} \quad (20)$$

Similarly,

$$\begin{aligned} r_s(t) y_e(t) &= \left[A \cos \left(\theta_r - \frac{\pi}{2} \right) \right] \left[E_e \cos \theta_e \right] \\ &= \frac{AE_e}{2} \left[\cos \left(\theta_r - \theta_e - \frac{\pi}{2} \right) + \cos \left(\theta_r + \theta_e - \frac{\pi}{2} \right) \right]. \end{aligned} \quad (21)$$

[2] "Determination of Signal Frequency, Phase and Amplitude from the Outputs of the Sequential Processor Fine Doppler Integrators," Bequaert and Manasse, MITRE, October 1962, AF19(628)-2390, TM-3410 (ESD-TDR-63-203).

Suppose we filter the high-frequency components and call the remaining signals $y_c(t)$ and $y_s(t)$, respectively; i.e.,

$$\begin{aligned} y_c(t) &= \frac{AE_e}{2} \cos(\theta_r - \theta_e), \\ y_s(t) &= \frac{AE_e}{2} \cos\left(\theta_r - \theta_e - \frac{\pi}{2}\right) = \frac{AE_e}{2} \sin(\theta_r - \theta_e). \end{aligned} \quad (22)$$

As stated before, we wish to obtain the phase measurements near the peak of the envelope; i.e., near $t'' = 0$. The reference phase θ_r can be expressed as a linear function of t'' . Now if we restrict ourselves to an interval I about $t'' = 0$, which is no larger than $\frac{1}{4}$ of the pulse width, θ_e can also be represented by a linear function of t'' .

Hence, the phase difference is a linear function of t'' , i.e., we let

$$\theta_r - \theta_e = ft'' + \phi, \quad (23)$$

where f and ϕ are functions of R and V . Using (17), we obtain, after some algebra,

$$f = \frac{V}{c+V} \left(\omega_0 + \frac{W}{2} \right), \quad (24)$$

$$\begin{aligned} \phi &= \omega_0 \left(\frac{2R}{c} + T \right) \pm \frac{T}{W} \left\{ \frac{2V}{c-V} \omega_0 \left[\frac{c^2+V^2}{c^2-V^2} \omega_0 + \left(1 - \frac{2Vc}{c^2-V^2} \right) \frac{W}{2} \right] \right. \\ &\quad \left. - \frac{2Vc}{(c^2-V^2)^2} \left[\left(c^2 - \frac{V^2}{3} \right) \omega_0^2 - \left(\frac{c^2}{3} \right) - V^2 \left(\frac{W}{2} \right)^2 - \frac{2}{3} \omega_0 W V c \right] \right\}. \end{aligned} \quad (25)$$

We assume the envelope E_e to be fairly constant* (say, $E_e = K$), in the interval I .

Then, using (23), we can write (22) as

$$\begin{aligned} y_c(t) &= \frac{AK}{2} \cos(ft'' + \phi) , \\ y_s(t) &= \frac{AK}{2} \sin(ft'' + \phi) . \end{aligned} \tag{26}$$

Averaging these functions over I , we obtain

$$Y_c = \frac{1}{I} \int_{-I/2}^{I/2} y_c(t) dt'' = \frac{AK}{I} \cos \phi \sin\left(\frac{fI}{2}\right) , \tag{27}$$

$$Y_s = \frac{1}{I} \int_{-I/2}^{I/2} y_s(t) dt'' = \frac{AK}{I} \sin \phi \sin\left(\frac{fI}{2}\right) . \tag{28}$$

Division of (28) by (27) yields

$$\frac{Y_s}{Y_c} = \tan \phi . \tag{29}$$

Note that this ratio is independent of the integration interval I , provided, of course, that the interval is symmetric about the line $t'' = 0$. (An error of ΔI in the determination of the center line $t'' = 0$ would not result in a large error if $\Delta I \ll I$, because we are dividing by I .) The phase angle ϕ thus measured is identical with what we would obtain if we were able to obtain a spot measurement of phase exactly at the peak of the envelope. It is very close (within 2 degrees in the worst case) to a spot measurement obtained anywhere

*In practice, the output function is passed through a limiter to make the envelope exactly flat in the desired interval and to eliminate dynamic range problems.

in the immediate vicinity of $t'' = 0$ ($|t''| \leq 1/8$ of the pulsewidth), because the time-varying term ft'' is a slowly varying function.

Let us now investigate how we might relate a sequence of phase measurements to the coefficient of the range polynomial of a given target. We shall use (25) as a description of each phase measurement. Rewriting (25) in an abbreviated form, we have

$$\phi = k_1 + k_2 R \pm F(V) , \quad (30)$$

where

$$k_1 \text{ and } k_2 \text{ are constants } \left(k_1 = \omega_0 T, k_2 = \frac{2\omega_0}{c} \right), \text{ and}$$

$F(V)$ is a smooth function of V , which can be very well represented by a polynomial in V over the region $0 \leq V \leq 10^4$ meters/second.

We emphasize again that R and V refer to the range and radial velocity, respectively, at the time that the target is being illuminated. Suppose this time were known. We could then plot two sets of points of measured phase vs time, one set corresponding to FM up, the other to FM down. The ambiguities for both sets of data can be resolved in the same way as is presently done with the MITRE Department D-82 radar. Each point would be plotted at the proper time of illumination. The two curves connecting these points would be smooth, since R and $F(V)$ are both smooth functions. The FM-up curve would deviate from the curve $k_1 + k_2 R$ by $+ F(V)$, and the FM-down curve would deviate from $k_1 + k_2 R$ by $- F(V)$. The average of the two curves would, therefore, be equal

to $k_1 + k_2 R$. Except for the additive constant k_1 and the scale factor k_2 , this average curve would then be identical to the range polynomial $R(t)$. Since k_1 and k_2 are known constants, we can consider the problem of determining the coefficients of our range polynomial as being virtually solved, provided that the illumination times can be determined.

Let us therefore turn to the problem of relating the time when the envelope peaks to the time when the pulse strikes the target. Let us refer to the former as t_M , the time of measurement, which, from (8), is equal to

$$t_M = \frac{2R}{c} + T \pm \frac{T}{W} \left(\frac{2V}{c-V} \right) \left[\omega_0 \left(\frac{c^2 + V^2}{c^2 - V^2} \right) + \frac{W}{2} \left(1 - \frac{2Vc}{c^2 - V^2} \right) \right] . \quad (31)$$

It is assumed that the pulse was transmitted at time $t = 0$. The time of illumination is given by $t_I = \frac{R}{c}$. Letting $G(V)$ be equal to the term following the \pm sign, we can write (31) as

$$t_M = 2 t_I + T \pm G(V) . \quad (32)$$

If we knew V , the radial velocity at time, t_I , exactly, we could evaluate $G(V)$ and determine t_I exactly. If we had a fairly good estimate of V we might be able to determine t_I to a high enough accuracy.

Now, $G(V)$ is a smooth function which for $0 \leq V \leq 10^4$ meters/second approximately linear with slope $\frac{T}{W} \left(\frac{2V}{c} - \omega_0 \right)$. We believe that we can estimate radial velocity from two consecutive target returns to an accuracy of 10 meters/second. The resulting error in the determination of t_I would then be

approximately $0.085 \mu\text{sec.}$, for the 1000:1 system, and $0.034 \mu\text{sec.}$ for the 10,000:1 system. (This error is of the same order of magnitude as the error in determining where the peak of the envelope occurs.) The corresponding maximum range error (for $V = 10^4$ meters/second) is less than 1 millimeter. Moreover, this type of error is "noise-like," and would be further reduced by smoothing, thus making it negligible.

We conclude, therefore, that it is quite feasible, in principle, to obtain accurate estimates of the pertinent target parameters by obtaining phase measurements on the compressed pulse. Note that the reasoning employed in arriving at this conclusion is valid, regardless of the degree of the range polynomial, $R(t)$. In fact, $R(t)$ need not even be a polynomial - any smooth function which allows interpolation between sample points would do. The same is true for the function $F(V)$, which is a measure of the deviation of the sampled phase function from $R(t)$. The only requirement on $F(V)$ is that it be a smooth function which is the same for FM up and for FM down.

Furthermore, it is not necessary to use a uniform repetition rate or any particular FM up-down pattern. The only requirement is that the up-down pattern allows the construction of two phase plots (one for FM-up, one for FM-down) which are valid over a common time interval. For the sake of being explicit, let us outline a procedure for relating the phase measurements to $R(t)$. Suppose N-coded pulses are transmitted at times T_1, T_2, \dots, T_N .

Let t_{Ij} be the time it takes the j^{th} pulse to reach the target [which would then be at range $R_j = R(t_j)$, where $t_j = T_j + t_{Ij}$]. Let t_{Mj} be the time at the peak of the j^{th} compressed pulse measured from T_j , and let ϕ_j be the phase measured at time $T_j + T_{Mj}$.

Figure 10 is a rough sketch of the relationship between the various quantities.

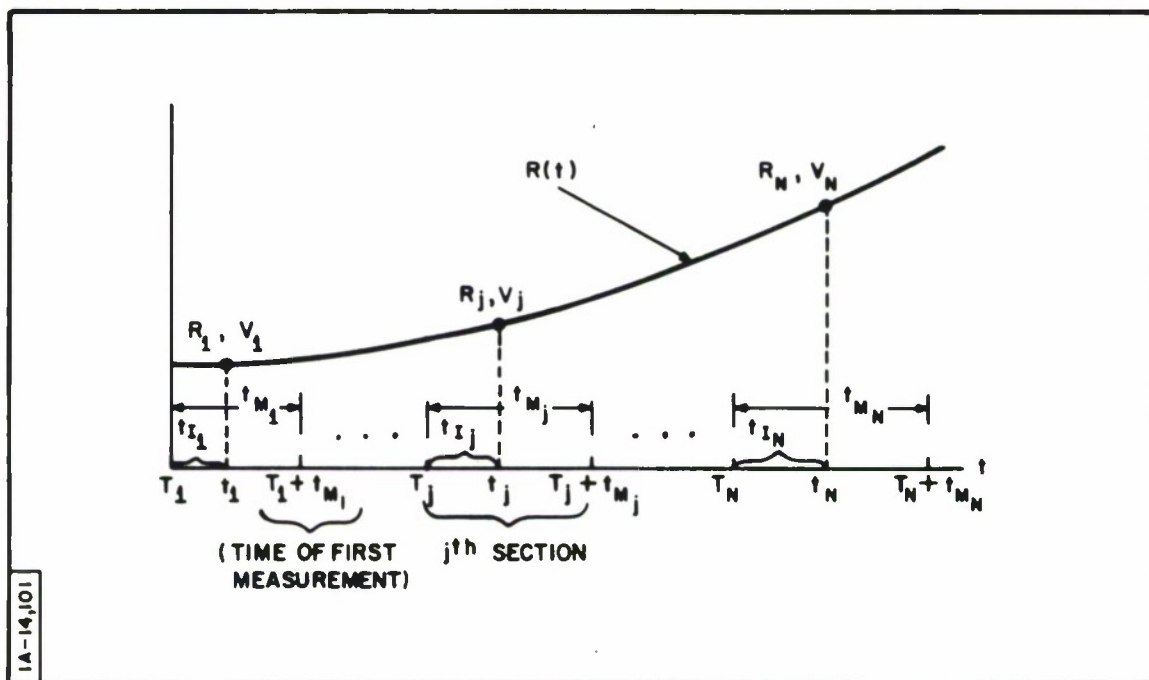


Fig. 10. Relationship Between Various Quantities

There are four pieces of data to be stored for each transmitted pulse. For the j^{th} pulse, these are T_j , T_{Mj} , ϕ_j and the mode of transmission (FM up or down).

Let us assume that a good estimate of V_1 is available. Using that value of V , $G(V)$ in (32) can be computed, and t_{I_1} can be obtained with the aid of (32). The measured value of phase can then be associated with the time, t_1 .

If this procedure is followed for all N pulses and if an up-down pattern is used which allows the construction of a curve for both FM up and FM down over a common region, one can obtain curves that look something like those sketched in Fig. 11.

Since the absolute value of phase cannot be determined from phase measurements alone, the position of the zero axis in Fig. 11 is arbitrary.

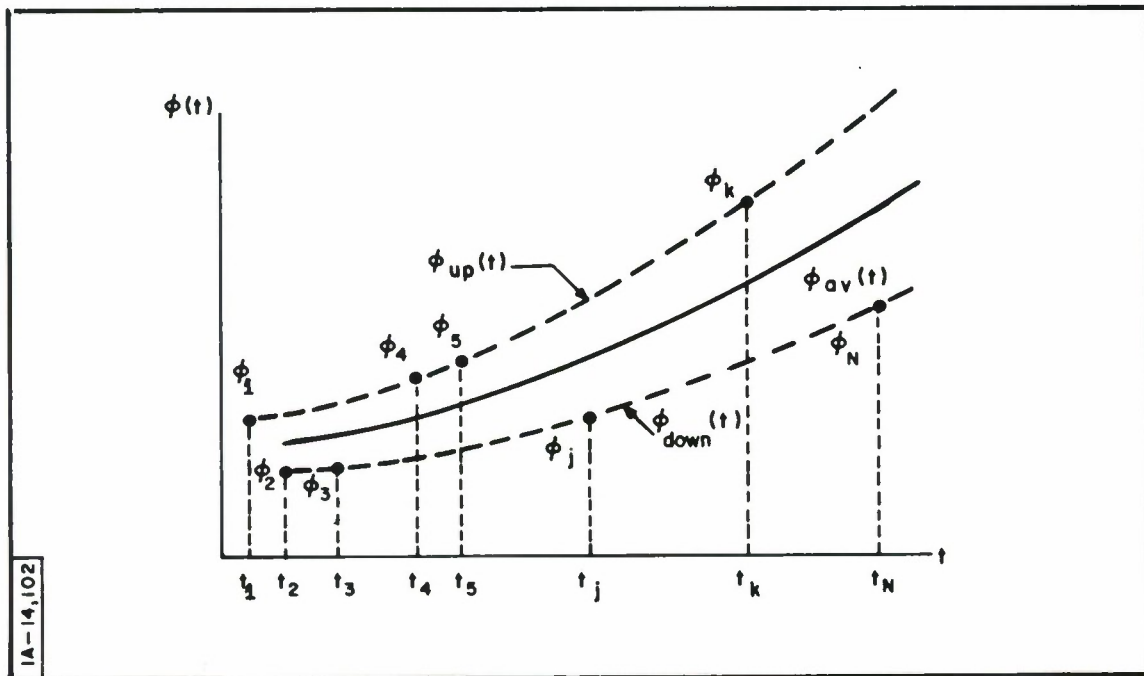


Fig. 11. Curves for FM Up and FM Down over a Common Region

In the figure, $\phi_{\text{up}}(t)$ and $\phi_{\text{down}}(t)$ denote the polynomials which have been fitted to the data points corresponding to FM up and FM down, respectively.

Letting $\phi_{\text{avg}}(t)$ be the average of these two curves; i. e., $\phi_{\text{avg}}(t) = \frac{1}{2} [\phi_{\text{up}}(t) + \phi_{\text{down}}(t)]$, we obtain, using Eq. (30),

$$\phi_{\text{avg}}(t) = \omega_0 T + \frac{2\omega_0}{c} R(t) . \quad (32)$$

If $\phi_{\text{avg}}(t)$ is described by a polynomial of degree n , i. e.,

$$\phi_{\text{avg}}(t) = a_0 + a_1 t + a_2 t^2 + \dots + a_n t^n ,$$

we can let

$$R(t) = b_0 + b_1 t + b_2 t^2 + \dots + b_n t^n ,$$

and solve for all the b coefficients, except b_0 , from the known a coefficients by letting

$$b_i = \frac{c}{2\omega_0} a_i , \quad i = 1, 2, \dots, n .$$

If $R(t)$ is known, except for the initial value b_0 , $V(t)$ is readily obtained by differentiating $R(t)$. The knowledge of $V(t)$, in turn, allows the determination of b_0 from the t_M measurements with the aid of Eq. (31). This finishes the problem if, as was assumed, we indeed had a good initial estimate of radial velocity. If such an estimate were not available, it could be obtained by

transmitting first an FM-down pulse at time T_1 , followed by an FM-up pulse at time T_2 (or vice versa), and observing the times t_{M_1} and t_{M_2} . From (31) we have, approximately,

$$\begin{aligned} t_{M_1} &= \frac{2R_1}{c} + T - \frac{T}{W} \left(\frac{2V_1}{c} \omega_0 \right), \\ t_{M_2} &= \frac{2R_2}{c} + T + \frac{T}{W} \left(\frac{2V_2}{c} \omega_0 \right). \end{aligned} \tag{33}$$

If we let \bar{V} be the average value of V_1 and V_2 , and let R_2 be approximately equal to $R_1 + \bar{V}(T_2 - T_1)$, (33) becomes

$$\begin{aligned} t_{M_1} &= \frac{2R_1}{c} + T - \frac{T}{W} \left(\frac{2\bar{V}}{c} \omega_0 \right), \\ t_{M_2} &= \frac{2R_1}{c} + T + \frac{2}{c} \bar{V} (T_2 - T_1) + \frac{T}{W} \left(\frac{2\bar{V}}{c} \omega_0 \right). \end{aligned}$$

Subtracting t_{M_1} from t_{M_2} , we obtain

$$t_{M_2} - t_{M_1} = \frac{2\bar{V}}{c} \left(T_2 - T_1 + \frac{2T}{W} \omega_0 \right),$$

so that

$$\bar{V} = \frac{c \left[t_{M_2} - t_{M_1} \right]}{2 \left(T_2 - T_1 + \frac{2T}{W} \omega_0 \right)}.$$

Note that the same procedure would apply if the exact expression (31) were used. We would only have to solve a more complicated expression for \bar{V} .

The value \bar{V} thus obtained is not very different from either V_1 or V_2 for any reasonable repetition frequency. Say a prf of 30 sec^{-1} is used, and consider a (very conservative) maximum radial target acceleration of $200 \text{ meters/second}^2$. Then V_2 is approximately equal to

$$V_2 = \left(V_1 + \frac{200}{30} \right) \text{ meters/second,}$$

and

$$\bar{V} \approx V_1 + 3.33 \text{ meters/second,}$$

which is well within the error of 10 meters/second we had previously considered.

Clearly, estimates of all successive values of V can be obtained by using the last available estimate. These estimates can be improved as more and more data points become available. We shall not go into any detailed discussion at this time of how the data processing might best be accomplished. It may be necessary to use an iterative technique to obtain a high enough accuracy in the estimation of the target parameters.

The next step in this theoretical investigation will be to consider what effect certain deviations from the idealized spectrum of the transmitted signal would have on the output function. In particular, we wish to know whether these effects are serious enough to invalidate our way of measuring and interpreting the data. We shall also consider the effects of noise, and what would be gained if we approached a matched filter more closely.

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| <p>Several expressions for the output time function of a linear-FM pulse-compression filter, which have recently been derived, (ESD-TDR-64-128) are compared and interpreted. Comparisons are made analytically as well as graphically with respect to several pertinent parameters. It is shown that if the phase and envelope measurements of signals returned from an actual target were to be interpreted as though the signal had undergone a Doppler shift (instead of a time dilation), a considerable error would result for high target velocities. The feasibility of using an all-pulse-compression system for obtaining accurate estimates of radar target parameters is demonstrated. Simple analytic expressions, which describe the phase and envelope quite well during the time of measurement, are presented, and a procedure for properly interpreting the data is indicated.</p> <p>(This Abstract is UNCLASSIFIED)</p> | | | |

14.

KEY WORDS

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